Table 2. Effect of trace metals on reproduction of the aster leafhopper, *M. fascifrons*

Trace metals deleted	Egg production* 1st generation	2nd generation	3rd generation
Fe	-		
Zn	+	+	-
Cu	+ +	+ + +	±
Mn	+ + +	+ + +	+ + +
Fe, Zn, Cu, Mn	±		
None	+ + +	+ + +	+ + +

* Observation on 10 females each diet; -, 0 eggs/female/day; ±, 0.1–0.9 eggs/female/day; +, 1.0–2.0 eggs/female/day; + +, 2.1–2.9 eggs/female/day; + + +, 3 or more eggs/female/day.

should be done through generations, rather than carried out in a short period of time. Carry-over by mothers and ionic contaminations in other components can not be overlooked when evaluating the nutritive values.

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A mathematical analysis of the disk-sphere transition of the human red cell

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Summary. By means of analytical calculations, an attempt is made to approximate the profile of the human red cell during the 'disk-sphere' transition induced by variation of the tonicity of the medium.

According to several authors[1,2] Cassini's oval is a plane curve of particular interest when one wishes to approximate the profile of any meridian section of the human red cell (HRC). It is well known that Cassini's oval is the locus of the points which satisfy the equation $(x^2+y^2+a^2)^2-4a^2x^2=m^4$, where a and m are 2 geometrical parameters. The purpose of this work is to attempt to find a law by which a and m may be related to a physical parameter T (tonicity) or, in other words, to supply the explicit expression of 2 functions of the following type:

(I) $$m=f(T); \quad a=g(T).$$

Methods. We hypothesized that T is linked to $\mathring{x}$ (equatorial radius) and to $\mathring{y}$ (polar radius) by means of simple relations of polynomial type, i.e.

(1) $$\mathring{x}=\sum_{i=0}^{2} A_i T^i; \quad \mathring{y}=\sum_{i=0}^{2} B_i T^i.$$

On the basis of the control values of $\mathring{x}$ and $\mathring{y}$, reported by Evans and Fung[3] for 3 T levels ($T_0=300$ mOsm/l; $T_1=217$ mOsm/l; $T_2=131$ mOsm/l), we were able to determine the coefficients A_i and B_i by means of the Hermite-Lagrange formula[4]. Given that:

(I bis) $$m^2=0.5\,(\mathring{x}^2+\mathring{y}^2); \quad a^2=0.5\,(\mathring{x}^2-\mathring{y}^2),$$

we finally obtained the functions:

(I ter) $$m=\left(\sum_{j=0}^{4}\alpha_j T^j\right)^{1/2}; \quad a=\left(\sum_{j=0}^{4}\beta_j T^j\right)^{1/2}$$

where
$$\alpha_0=\frac{1}{2}(A_0^2+B_0^2) \qquad \beta_0=\frac{1}{2}(A_0^2-B_0^2)$$
$$\alpha_1=(A_0A_1+B_0B_1) \qquad \beta_1=(A_0A_1-B_0B_1)$$
$$\alpha_2=(A_0A_2+B_0B_2)+\frac{1}{2}(A_1^2+B_1^2) \qquad \beta_2=(A_0A_2-B_0B_2)+\frac{1}{2}(A_1^2-B_1^2)$$
$$\alpha_3=(A_1A_2+B_1B_2) \qquad \beta_3=(A_1A_2-B_1B_2)$$
$$\alpha_4=\frac{1}{2}(A_2^2+B_2^2) \qquad \beta_4=\frac{1}{2}(A_2^2-B_2^2)$$

By varying T in the experimentally significant range between 300 mOsm/l and 131 mOsm/l, we determined the corresponding configurations of Cassini's oval, and numerical values of the surface area and volume of the 'mathematical cell' produced by the rotation of the curves around the polar axis. We used the following formulae:

$$\text{volume} = 4\pi\int_0^{\mathring{x}} x f(x)\,dx,$$

$$\text{surface area} = 4\pi\int_0^{a} x\sqrt{1+[df(x)/dx]^2}\,dx + 4\pi\int_{f(a)}^{0} x\sqrt{1+[dh(y)/dy]^2}\,dy,$$

where f(x) is the equation of Cassini's oval explicated with respect to y, and h(y) is the inverse function in the interval $(a, \mathring{x})$. The inverse function and the particular limits of integration were chosen in order to eliminate any possible singularity of the surface area integral. The numerical calculation of the integrals was made using an IBM computer 370/158 (Fortran IV language) on the basis of the Gauss 10 points quadrate formula[5]. Profiles were obtained using the CALCOMP 925/1036 unit.

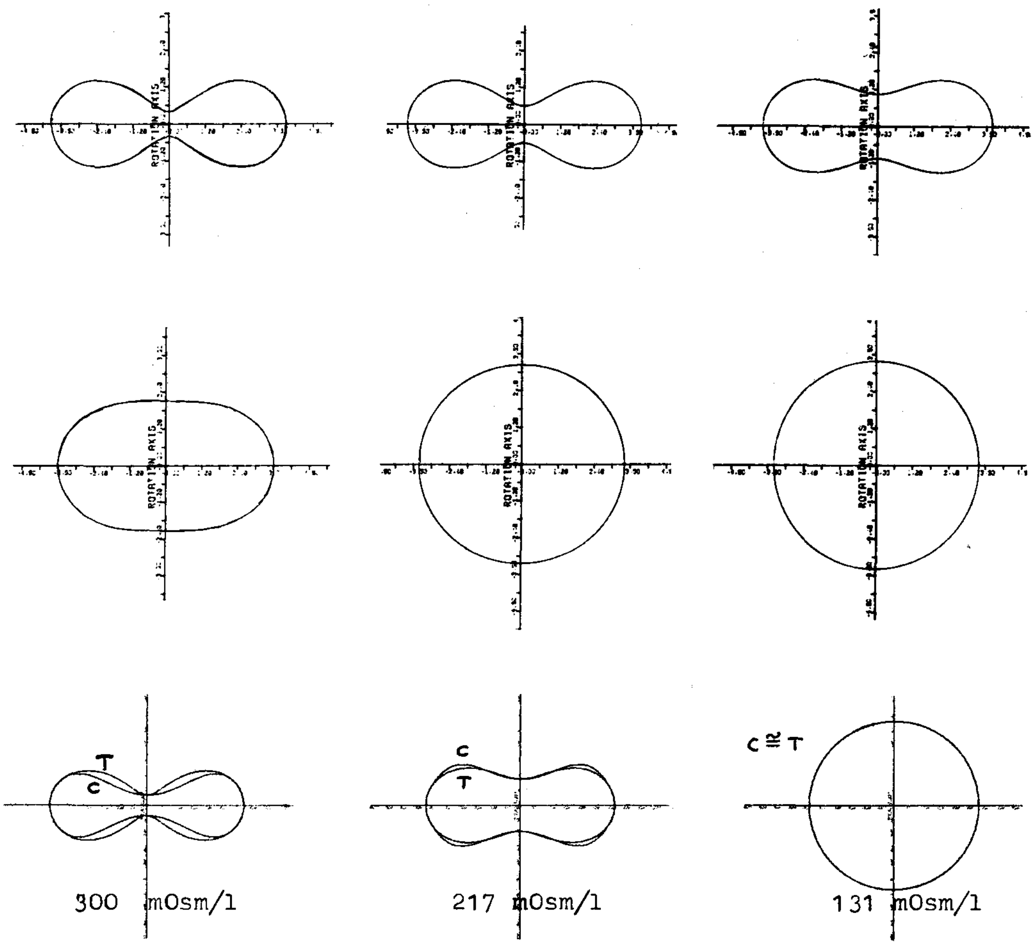
Results. The results we obtained are reported in tables 1 and 2.

Table 1. For explanation see text.

Stage	Tonicity (mOsm/l)	$\mathring{x}$ distance (μm)	$\mathring{y}$ distance (μm)	Surface area (μm²)	Volume (μm³)
* 1	300	3.910	0.404	135.318	106.887
* 2	250	3.873	0.604	132.744	106.790
3	230	3.831	0.844	130.286	108.001
* 4	217	3,795	1.050	128.698	109.897
* 5	170	3.610	2.118	128.486	128.571
6	140	3.446	3.065	138.853	153.364
* 7	135	3.415	3.243	141.785	158.700
8	133	3.403	3.316	143.067	160.944
9	132	3.396	3.353	143.733	162.090
*10	131	3.390	3.390	144.415	163.251
* 1	300	3.910	0.404	135.000	94.000
* 4	217	3.795	1.050	135.000	116.000
*10	131	3.390	3.390	145.000	164.000

Table 2.

Above: Theoretical profiles (profiles of the 'mathematical cells') are reported as outlined by computer. The table 2 demonstrates the stages 1, 2, 4, 5, 7, 10, labeled with * in table 1. Below: Superimposition of the 3 control profiles (stages 1, 4, 10, respectively) on the corresponding theoretical ones (C=control profile; T=theoretical profile).



In the top-part our results are reported, in the bottom-part the control values from Evans and Fung[3] on the basis of measurements of erythrocyte geometry. It is necessary to emphasize that the control values concern only 3 of our results, because Evans and Fung[3] studied the erythrocyte geometry only in three different medium tonicities.
We have reported in table 1 also the values of $\hat{x}$ and $\hat{y}$ which agree 'by construction' with the control values of Evans and Fung[3].

Discussion. The present work concerns an analytical procedure, extraordinarily simplified, by means of which the Cassini's oval (a model for the human erythrocyte) may be linked to value of medium tonicity.
Few of our results may be compared to those of Evans and Fung[3], because the geometry of the red cell has been investigated by the above-mentioned authors with regard to 3 conditions of the medium tonicity only (i.e. T_0=300 mOsm/l; T_1=217 mOsm/l; T_2=131 mOsm/l).
It appears a satisfactory agreement between our model and its prototype (red cell) with regard to the initial and terminal stage of the osmotic swelling; in the intermediate stage, however, the data are far from coincident.
As for the surface area, our results might suggest the existence of a minimum value; this fact, however, does not indicate an actual inversion of the rheological behaviour of the red cell's membrane, being merely an handicap of our analysis according to which only $\hat{x}$ and $\hat{y}$ were kept identical to those of the true cell.

In fact, by using a biparametrical model, as the Cassini's oval, it is not possible to obtain the concordance between the theoretical and actual data concerning simultaneously the axial distances (i.e. $\hat{x}$ and $\hat{y}$) and the surface area (or volume); the imposition of axial concordance limits the surface area concordance to narrow fields, closed to the extremes of the range of the medium tonicity. This restrictive disadvantage is, obviously, overcome by using a geometrical model having more than 2 parameters; in such case, however, the link between tonicity and shape coefficients is anything but easy. The Cassini's oval (which, on the other hand, is not suitable for red cells, having sphericity index less than 0.78875) is, of course, not very reliable because of its scarce morphological pliability, but it is rather helpful for obtaining first approximate information about the erythrocyte geometry during the gradual, osmotical transformation due to variation of the medium tonicity.

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